Differential Equations and the AP Exam

Differential Equations are an important part of the AP Calculus Exam. While they sometimes appear as multiple choice questions, they have appeared frequently in the free response section of the AP Exam. When they do appear as free response they have tended to follow a pattern which you should make sure that you’re familiar with.

I will borrow a Diff-E-Q from the 2005 AP Exam to use as an example:



**Step 1 – Separate**

You must start by viewing the top and the bottom of as two separate entities and splitting them up like this:

Note that the y is grouped with the “dy” and the -2x is with the “dx”. This step is called **separating**.

**Step 2 – Integrate (or anti-differentiate)**

Next, you will find the indefinite integral (anti-derivative) of both sides:

[don’t worry about why I wrote the “C’s” this way]

**Step 3 – Oh Say Can You “C”?**

The constant of integration – “C” – is one of the easiest points to earn on the AP Exam. You will see this later when we go through a scoring guide for this type of question. Once you take the anti-derivative of both sides you should combine the constants on one side:

You’re allowed to do this since the “C’s” are arbitrary at this point anyways.

**Step 4 – Using the Initial Condition**

The result that we got above is just a couple of algebraic steps away from being “solved” for a general solution. Remember that there are many functions that will have the derivative that we started with.

However, you will typically be asked to find a “particular” solution, that is, one that particular function that has the derivative. To find this you need an initial condition:



This means that we want the particular function with derivative that will go through the point (1 , -1). There is only one such function, and we find it by plugging in to x and y:

**Step 5 – Finding the Particular Solution**

Now that we have a value for “C”, we can solve the function for “y” and get our particular solution:

Now we have to figure out whether we are on the “+” part or the “ - “ part of the function. We do this using our initial condition. Since the initial condition is (1 , -1), we are on the negative part.

Therefore, our final particular solution is:

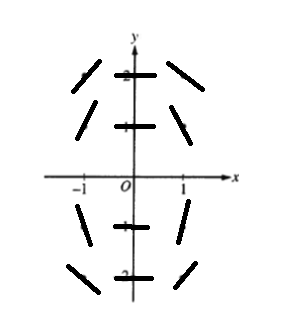
You may have noted (correctly) that we could have solved the equation for y first and then plugged in our initial condition. That would have worked as well. I didn’t do it because that requires that you play some games with the constant of integration. We’ll see one later where we’ll want to solve for y first.

**Slope Fields – a Graphical Representation of a Function**

A concept that is grouped together with Diff-E-Q’s is the idea of a slope field. A slope field is a series of tiny slopes based on a Diff-E-Q. They give a rough idea of the shape of the solution function. This is another good place to distinguish between the “general” and “particular” solutions.

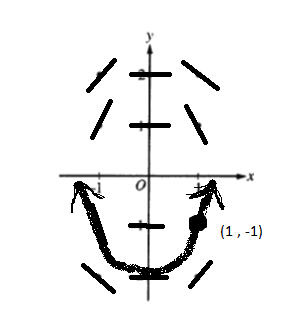


At each point, draw a short line that has the slope you find by plugging the x-coordinate and y-coordinate into the Diff-E-Q.



This is just a representation of the general solution. Clearly, this isn’t actually a function (it fails the vertical line test, for one thing).

Now, let’s add the initial condition to the graph:



**Guided Practice**

Here are two separable Diff-E-Q’s to try out. We’ll add in slope fields later.



